Homework Answers: Probability

1) Your answer

2) Your answer

3) a. Define the events:

1_O = the Person 1 has blood type O. (These are capital Os, not zeros.)
2_O = the Person 2 has blood type O.

Assuming that Person 1 has a blood type that is **independent** of Person 2's blood type, then the probability that both have blood type O is:

P(1_O and 2_O) = P(1_O) * P(2_O) = .46 * .46 = 0.2116.

4) This homework problem is a tough one. This kind of problem is designed to challenge you. It requires you to combine the AND and OR rules in the same calculation. You **WILL NOT** be asked to do anything this complicated on the test. So, long as you understand how to apply the AND and OR rules to separate problems, you will be prepared for the test. However, working through and understanding more difficult problems, should prepare you even more for simpler problems that you will see on the test.

Similarly, define the events:

1_A = the Person 1 has blood type A.
2_A = the Person 2 has blood type A.

1_B = the Person 1 has blood type B.
2_B = the Person 2 has blood type B.

1_{AB} = the Person 1 has blood type AB.
2_{AB} = the Person 2 has blood type AB.

Then, the ways for both to have the same blood type include: (1_O and 2_O), (1_A and 2_A), (1_B and 2_B), (1_{AB} and 2_{AB})

Assume that Person 1's blood type is independent of Person 2's blood type. We then can compute the probabilities of these events like we did for (1_O and 2_O), but now we have to add after multiplying.

We use the **multiplication rule** because we are looking at joint probability for two independent events. We use the **addition rule** because we are looking at more than one possible outcome for mutually exclusive (*disjoint*-having no outcomes in common) events – couple would either be AA, or OO, or AB or BB, but could not be both AA and OO or AB and BB, and so forth.

P (both same type) = P (1_O **and** 2_O) + P (1_A **and** 1_A) + P (1_B **and** 2_B) + P (1_{AB} **and** 2_{AB})

= (.46 * .46) + (.40 * .40) + (.10 * .10) + (.04 * .04)

= 0.3832.

These assumptions are reasonable provided that people do not look for mates that have similar blood types to them. For example, if people look for someone of the same race or ethnicity, and if blood types are in different proportions for different ethnicities (which they are), then Person 1's blood type will NOT be independent of Person 2's blood type.

For example, assuming men with blood type AB look for women that also have AB, then knowing that a man has blood type AB means it is more likely that the woman he is with also will have blood type AB. Now, the probability of one event depends on the other or is conditional. So the above calculations would yield an incorrect answer.

Moral of the story: assumptions of independence need to be checked thoroughly before they are accepted.

5) Your answer

6) Your answer

7) a. If 28% of rabbits have long hair and there are five rabbits in the litter, then .28 * 5 = 1.4. Would expect 1 to 2 rabbits with long hair in this litter.

b. Same answer as previous. Just asked the question in a different way. Probability of one rabbit having short hair = .72. Probability that all the rabbits have short hair (.72*.72*.72*.72*.72 = 0.1935)

8) Your answer

9)

a. Both sequences are equally likely. Your sequence of flips = P(HTHHTHTTTH) = P(H) * P(T) * P(H) * P(H) * P(T) * P(H) * P(T) * P(T) * P(T) * P(H) = .5 * .5 * .5 * .5 * .5 *.5 * .5 * .5 * .5 * .5

Your friend's sequence of flips = Pr(HHHHHHHTTT) = P(H) * P(H) * P(H) * P(H) * P(H) * P(H) * P(H) * P(T) * P(T) * P(T) = .5 * .5 * .5 * .5 * .5 *.5 * .5 * .5 * .5 * .5

Because your friend's sequence seems to have a pattern, you might think that the probabilistic forces at work are different in her case. However, one outcome (or a sequence of outcomes) does not change our predictions about the next, independent outcome.

b. A "heuristic" is a mental shortcut—an application of old understanding (strategies) to solving or understanding new problems. The **representativeness heuristic** is the tendency to judge the frequency or likelihood of an event by the extent to which it resembles the typical case.

For example, in a series of 10 coin tosses, most people judge the series HHTTHTHTTH to be more likely that the series HHHHHHHHHH (where H is heads and T is tails), even though both series are equally likely. The reason is that the first series looks more "random" than the second series. That is, it "represents" our idea of what a **"random"** series should look like. Right? But what does random even mean? For most people, randomness is chaos, unpredictability. But really…there may not even be such a thing as randomness. Bahhh!!?? Read on.

We get into this type of thinking when we are trying to predict a **single event** that is just as likely as other events in a problem space. A coin toss is a good example. The accuracy of our predictions will be lower when events in a problem space are all about equally likely—as are heads and tails. That's what leads us into thoughts ("I don't know if it will be heads or tails on the next flip") about randomness. But again, over the long run—say, 1,000 unbiased tosses of the coin—our prediction of 50% heads and %50 tails will be supported almost perfectly. So what…it's no longer random when we do it a bunch of times? That doesn't make sense.

Really, probability isn't about unpredictable, random chaos of individual events or short sequences of events. **Probability is about trends that we see over the long run**. In other words, probability derives not from a single instance or even brief sequence of events, but from an infinite number of sequences that are infinitely long. In theory, that's where probability comes from. The problem is that most of us see only small parts of events or have limited knowledge. So, many events feel quite random.

Here's another way to think about it: The more we know about an event, the less random it seems. Take for example, the weather. For most of us, the weather can seem very random at times. But what about a meteorologist or geographer who understands the relationships between humidity, altitude, location, air pressure, global warming, etc.? This person would be far better at predicting weather and see it as significantly less random than the rest of us. If we had this knowledge, we could predict the weather with much greater accuracy. As a result, the event would feel "less random." It's not that the probability of the event has changed—just our understanding of it.

As many in the field of statistics maintain, randomness isn't a fact about an event. It's a fact about our knowledge or understanding of an event. So, randomness, as most people think about it, may not even really exist. It's just a convenient shortcut (heuristic) for discussing the occurrence of events, the causes of which, we don't fully understand.

c. Answers vary