APPENDIX 1

BRIEF MATH–ALGEBRA REVIEW

Let's face it: There's a lot of variability in the mathematical sophistication undergraduates bring to their first statistics courses. From our experience, preparation ranges from avoidance of any college math to completion of an advanced college calculus course. If your background qualifies you for the calculus end of the range, read no further; this appendix is not for you. On the other hand, if you've avoided college math before this, read on. The material in this appendix is designed to refresh your memory of topics long forgotten and to prepare you for the very basic mathematics presented elsewhere in this text.

SYMBOLS

Symbol	*Read*	*Example*
=	equals	$7 = 7$
≠	is not equal to	$6 \neq 37$
>	greater than	$5 > 2$
<	less than	$2 < 5$
≥	greater than or equal to	$p \geq .05$
≤	less than or equal to	$p \leq .01$
+	plus	$3 + 3 = 6$
–	minus	$3 - 3 = 0$
× or ( )( ) or ·	times	$3 \times 3 = 9$ $(3)(3) = 9$ $3 \cdot 3 = 9$

Symbol	*Read*	*Example*
÷ or /	divided by	$6 \div 3 = 2$ $6/3 = 2$
$(\ )^2$	square	$(2)^2 = 2^2 = 4$
$\sqrt{\ }$	square root	$\sqrt{4} = 2$
$\lvert\ \rvert$	absolute value of (the number without regard to sign)	$\lvert -5 \rvert = 5$ $\lvert 5 \rvert = 5$
$\pm$	plus or minus	6 ± 5 is $6 + 5 = 11$ and $6 - 5 = 1$

ARITHMETIC OPERATIONS

Addition

Numbers may be added in any order or combination.

Example:

$$8 + 4 + 3 = 3 + 4 + 8 = (3 + 4) + 8 = 15$$

Addition of Negative Numbers

Add as though the numbers were positive; then put a negative sign before the result.

Example:

$$(-1) + (-3) + (-5) + (-7) = -(1 + 3 + 5 + 7) = -16$$

Subtraction

Subtraction is equivalent to the addition of a negative number.

Example:

$$12 - 3 = 12 + (-3) = 9$$

As with addition, numbers may be subtracted in any order or combination.

Example:

$$27 - 4 - 12 = 27 + (-4) + (-12) = 27 + [(-4) + (-12)] = -4 + (-12) + 27 = 11$$

Adding a Combination of Positive and Negative Numbers

Sum the positive and negative numbers separately, take the difference between the sums, then put the sign of the larger sum on the result.

Examples:

$(+5) + (+3) + (+6) + (-2) + (-5) + (-15) = +14 + (-22) = -8$ (The difference between 22 and 14 is 8, and the sign of the larger difference is –.)

$(+5) + (+10) + (-3) + (-5) = +15 + (-8) = 7$

Subtracting Negative Numbers

Change the sign of the negative number to be subtracted and add.

Examples:

$10 - (-5) = 10 + 5 = 15$
$-15 - (-6) = -15 + 6 = -9$

Multiplication and Division

In multiplication and division, the result is positive if all of the terms are positive or if there is an even number of negative terms. The result is negative if there is an odd number of negative terms.

Examples:

$(6)(3) = +18$
$(-6)(-3) = +18$
$(-6)(3) = -18$ or $(6)(-3) = -18$

$$\frac{-6}{-3} = +2$$

$$\frac{6}{3} = +2$$

$$\frac{6}{-3} = -2$$

$$\frac{-6}{3} = -2$$

As with addition and subtraction, order of computation makes no difference.

Example:

$(6)(-3)(-4) = (-4)(-3)(6) = 72$

ORDER OF OPERATIONS

Work from the inside out; that is, do everything inside parentheses or brackets first. Within an expression, follow the order of operations summarized in the mnemonic "Follow My Dear Aunt Sally."

1. Follow = Functional operations such as square or square root
2. My Dear = Multiplication and Division
3. Aunt Sally = Addition and Subtraction

Example 1

$$6(5 - 2^2) + \frac{13\sqrt{25}}{8} = 6(5 - 4) + \frac{(13)(5)}{8} = 6(1) + \frac{65}{8} = 6 + 8.13 = 14.13$$

Example 2

$$\left[\frac{(.9)(5)}{4}\right]X + \left[20 - \left(\frac{(.9)(5)}{4}\right)(12)\right]$$

Do everything inside parentheses or brackets first. Multiply, then divide:

$$\left(\frac{4.5}{4}\right)X + \left[20 - \left(\frac{4.5}{4}\right)(12)\right]$$

$$1.13X + [20 - (1.13)(12)]$$

Multiply before adding:

$$1.13X + (20 - 13.56) = 1.13X + 6.44$$

WORKING WITH FRACTIONS

In these days of the pocket calculator, the simplest way to work with fractions is to convert them to decimals and carry out the operations specified by the equation. To convert a fraction to a decimal, divide the numerator (top part of the fraction) by the denominator (bottom part of the fraction).

Examples:

$$\frac{1}{15} + \frac{1}{17} = 0.07 + 0.06 = 0.13$$

$$\left(\frac{3}{19}\right)\left(\frac{7}{16}\right) = (0.16)(0.44) = 0.07$$

Nevertheless, there are times at which it is more convenient or it is necessary (such as when your calculator breaks) to work with fractions. In these situations, there are several rules, specific to fractions, that must be followed.

Addition or Subtraction of Fractions

In adding or subtracting fractions, first express each fraction in terms of the least common denominator. Then add (or subtract) the numerators and place the result over the common denominator.

Examples:

$$\frac{1}{8} + \frac{1}{2} + \frac{1}{4} = \frac{1}{8} + \frac{4}{8} + \frac{2}{8} = \frac{7}{8}$$

$$\frac{11}{12} - \frac{1}{6} - \frac{1}{3} = \frac{11}{12} - \frac{2}{12} - \frac{4}{12} = \frac{5}{12}$$

Multiplication of Fractions

In multiplying fractions, find the product of the numerators and divide by the product of the denominators.

Examples:

$$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15}$$

$$\frac{1}{2} \times \frac{3}{7} \times \frac{3}{5} = \frac{1 \times 3 \times 3}{2 \times 7 \times 5} = \frac{9}{70}$$

Multiplying a Whole Number by a Fraction

In multiplying fractions and whole numbers, multiply the numerator of the fraction(s) by the whole number(s) and divide by the denominator.

Examples:

$$\left(\frac{1}{9}\right)(5) = \frac{(1)(5)}{9} = \frac{5}{9}$$

$$\left(\frac{1}{7}\right)(4)(3) = \frac{(1)(4)(3)}{7} = \frac{12}{7}$$

$$\left(\frac{4}{5}\right)\left(\frac{1}{3}\right)(7) = \frac{(4)(1)(7)}{(5)(3)} = \frac{28}{15}$$

SOLVING EQUATIONS WITH ONE UNKNOWN

You frequently will be required to solve equations with one unknown. These are quite simple and usually involve performing arithmetic steps to isolate the unknown on one side of the equation and then simplifying the other side of the equation. The specific steps required are different for each problem. The principle to remember is that whatever you do on one side of the equation must also be done on the other side of the equation. Examples of the basic types of problems you may encounter will illustrate the procedures involved.

Solving for X by addition:

$$X - 7 = 4$$
$$X - 7 + 7 = 4 + 7 \quad \text{(add 7 to both sides)}$$
$$X = 11$$

Solving for X by subtraction:

$$X + 6 = 10$$
$$X + 6 - 6 = 10 - 6 \quad \text{(subtract 6 from both sides)}$$
$$X = 4$$

Solving for X by division:

$$3X = 15$$
$$\frac{3X}{3} = \frac{15}{3} \quad \text{(divide both sides by 3)}$$
$$X = 5$$

Solving for X by multiplication:

$$\frac{X}{7} = 4$$
$$(7)\frac{X}{7} = (7)4 \quad \text{(multiply both sides by 7)}$$
$$X = 28$$

Solving for X by a combination of procedures:

$$4X + 2 = 18$$
$$4X + 2 - 2 = 18 - 2 \quad \text{(subtract 2 from both sides)}$$
$$4X = 16$$
$$\frac{4X}{4} = \frac{16}{4} \quad \text{(divide both sides by 4)}$$
$$X = 4$$

$$\frac{X - 10}{3} = 2$$
$$(3)\left(\frac{X - 10}{3}\right) = (3)2 \quad \text{(multiply both sides by 3)}$$
$$X - 10 = 6$$
$$X - 10 + 10 = 6 + 10 \quad \text{(add 10 to both sides)}$$
$$X = 16$$

EXERCISES

1. $\dfrac{3 + (13 - 3^2)}{(4)(6 - 3)(5 + 2^2)} = ?$

2. $\dfrac{1}{10} + \dfrac{5}{17} + \left(\dfrac{6}{23}\right)\left(\dfrac{5}{11}\right) = ?$

3. $\left(\frac{15}{27}\right)\left(\frac{3^2}{13^2}\right) = ?$

4. $13 + (-7) + 14 + (-3)^2 = ?$

5. $15 - (-10) = ?$

6. $\left|-27\right| = ?$

7. $(-11)^2 = ?$

8. $(-10)(13) = ?$

9. $\frac{10\sqrt{5}}{\sqrt{\frac{275}{12} - 4^2}} = ?$

10. $\frac{(8)(7^2) + 1/3(7 + 6^2)}{(10 + 11 - 2)(6 - 3)} = ?$

11. $\frac{17}{10 - 15} = ?$

12. $\frac{3X + 14}{5} = 10 \qquad X = ?$

13. $\frac{2}{3}X - 3 = 11 \qquad X = ?$

14. $\frac{3(X + 2)}{7} = 6 \qquad X = ?$

15. $7X - 15 = -1 \qquad X = ?$