Chapter 8: Probability

Objectives for This Chapter

- Be able to define and discuss intuitively probability in our everyday lives.
- Understand and be able to use some basic rules from probability theory.
- Understand the differences between the addition and multiplication rules.
- Distinguish between independent and dependent probability events.
- Understand the basics of conditional probability.
- Distinguish among the different forms of subjective probability.
- Be able to perform requisite calculations.

What do you mean I already know about probability, Chance and Randomness?

Most of us intuitively use or at least encounter the idea of probability every day of our lives. For example, we may listen to the radio weather report and hear that the probability of rain is 20%. On the basis of this information, we may decide not to take an umbrella or raincoat with us, or, if we are more cautious, we may elect to be safe rather than sorry. If we decide to reduce our intake of red meat, it may be because we have heard that the probability of heart disease is lower for people with low-fat diets.

In other words, we are constantly assessing probabilities, whether we do it consciously or not, whether we are right or wrong. Many of our actions are based in part on these assessments--that is, whether we feel an event is likely or unlikely to occur. Our feelings about the probability of an event's occurrence may be based on pure speculation or intuition, on previous experience with similar events, or on a knowledge of theoretical outcomes.

Probabilities come in many different disguises. Some of the terms people use for probability are *chance, randomness, likelihood, odds, percentage,* and *proportion*. There are actually subtle differences among these terms. The term chance is an important one to grasp in "getting our heads around" the concept of probability. Typically, when we are talking about chance, we are referring to individual events. For example, "What are the chances of me winning the lottery?" Or how about, "Because cancer accounts for 1 out of 4 deaths, there is a 25% chance that I will die from cancer?" You can signify a chance with a percent (80%), a proportion (0.80), a fraction (8/10) or a less precise but still useful word (such as "likely"). However you calculate it, **chance** *is a description of our ability to predict a single outcome* (e.g., a single person, a single role of the die, a single toss of the coin).

But single outcomes are not the "end game" in science. Scientists like to make predictions that account for lots and lots of outcomes. The bottom line of all probability analyses is that they revolve around **long-term chance**. And this is extremely important and valuable. When you're looking at a sequence of events, you're able to see more (i.e., patterns in the data) than if you looked at one event in isolation. More on this, below.

The term **random** is another elusive one. According to some, randomness refers to chaos, unpredictability, and uncertainty. Many others (including scientists and researchers) maintain that random events are those events for which all outcomes are equally likely--as when one rolls a die (each side of the die has an equal chance of showing face-up) or randomly selects a sample of participants for an experiment. In everyday conversation, the concept of random is frequently equated with "fairness." Still, many strongly associate randomness with something that is peculiarly out of place or weird ("My social life is so random!"). Truly, there is no concensus on exactly what "random" means.

Kaplan, Rogness and Fisher (2012) investigated some of the linguistic difficulties in getting everyone on board with the same statistical definition for the concept of random. Ironically, even these researchers couldn't stick to the same definition throughout their research paper. Nonetheless, they had some great insights. They proposed that seeming disagreements on the concept of random, may have much to do with how the concept is used.

For example, suppose you have a room in which there are five men and 10 women. We will further assume that you are blindfolded. [*C'mon, Dr. B. I don't go in for that sort of thing.*] Now, two things. First, selecting any of the people in the room is a matter of chance and each person does have a probability of 1 in 15 of being chosen. Secondly, the outcome of choosing a man versus a woman is different; that is, the likelihood of choosing a man is about 33%, whereas that of choosing a woman is about 66%. Within this scenario, we have two random processes: 1) *random sampling*, wherein all outcomes are equally likely, and 2) *predictability,* which must be ascertained over theoretically infinite trials.*

For the purposes of this course, we will agree [*pardon my fascisim*] that it is a combination of the above applications (i.e., random sampling and predictability) that brings us closest to the kind of randomness most relevant to statistical analyses. We would say then, that **randomness** is when *the outcomes of individual events are unknown, but there is, nonetheless, a pattern of outcomes that becomes apparent and can be predicted over a large number of repetitions* (Moore, et al., 2013). And most statisticians believe that given enough time and study, we can find the patterns in just about anything. As one of Sir Arthur Conan Doyle's characters (can't remember if it was Sherlock or Mycroft Holmes) once quipped regarding coincidence and chaos, "The universe is rarely so lazy."

Elementary, my dear Watson!

*Suppose that in the scenario above, we made nine selections of individuals. Further suppose that throughout all of these selections, there were always 15 people in the room and the gender ratio of males (m) to females (f) was always 1:2. What if your six selections resulted in the following pattern: mffmffmff ? Predicting m on the tenth selection is too easy--too predictable. There's very little uncertainty. We would probably conclude that there is some kind of bias built in to the selection process, such that our prediction would not be based on probability but instead, a simple understanding of the sequence. There is really no room to talk about "randomness" in such a case. Conversely, irregularity within large numbers of trials is evidence of randomness, and it forces us to probabilistically identify patterns and predict future outcomes.

References

Moore, D. S. , Notz, W.I., & Fligner, M.A. (2013) The Basic Practice of Statistics (6th ed.). W. F. Freeman and Company, New York.

Kaplan, J.J., Rogness, N. & Fisher, D. (2012). Lexical ambiguity: Making a case against spread. Teaching Statistics, 34(2), 56 – 60. DOI: 10.1111/j.1467-9639.2011.00477.x

What are some important categories of probability?

Theoretical probability describes a sample space in which all possible outcomes are equally likely. It is the way events are supposed to work--in theory, or in terms of *formal* probability. In this realm, situations and events are perfect. In this realm, we have almost absolute control over our variables. Our dice are perfectly formed, and the rolls of the dice are totally unbiased. The deck of cards is perfectly shuffled each time the poker game begins. As a consequence, theoretical or formal probabilities can be derived with great precision. But as you may have guessed, most of our world does not operate according to perfect probability.

In fact, most of the time, our lives and decision making fall under the category of ***real-world*** or *empirical* probability; that is, *historical or empirical assessments of chance in real life.*

Anyone who can count can become an expert in real-world probability. This kind of probability is the basis for many assessments of chance that affect our lives. Insurance companies base their rates, to a large extent on real-world probabilities. For example, if your home is in one of the heavy tornado states, then your home insurance rates will be high, because insurance companies have collected data over many years indicating that homes in this part of the country receive windstorm damage more often than homes in other areas. Similarly, regardless of the driving record of any particular teenage male, all teenage male drivers pay much higher car insurance rates than other classes of drivers because of the accident rates of teenage male drivers as a group. What are some other factors that might affect a driver's insurance rate? How do these factors relate to probability?

Other results of real-world probability assessment include Walmart's decisions about what to sell and where to sell it, doctors' decisions about how to treat a difficult cancer, and the batting averages of baseball players. In each of these instances, probabilities are based on past behavior and counting. Statisticians become concerned when theoretical probabilities and empirical probabilities do not match up well.

The last category of probability is ***subjective probability***, which is *a probabilistic prediction based on an individual's personal judgment, not on mathematical calculations.* Subjective probability is usually quite inaccurate and can lead to some irrational decisions. More about this, later in this chapter.

How do we calculate the probability of a single event?

If you roll a six-sided die, there are six possible outcomes, and each of these outcomes is equally likely. A six is as likely to come up as a three, and likewise for the other four sides of the die. Since there are six possible outcomes, the probability is 1/6. The outcome about which we are concerned (a one or a six coming up) is called a *favorable outcome*. I will sometimes refer to these as *desireable outcomes* or *events*. Given that all outcomes are equally likely, we can compute the **probability of an event** using the formula:

$$\text{probability} = \frac{\text{Number of favorable outcomes}}{\text{Number of possible equally-likely outcomes}}$$

Sometimes, I will refer to *all possible outcomes* as the *probability space* or *sample space*. In this case there is one favorable outcome and six possible outcomes. So the probability of throwing a six is 1/6. Don't be misled by the use of the term "favorable," by the way. You should understand it in the sense of *favorable to the event in question happening*. That event might not be favorable to your well-being. You might be betting on a three, for example.

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Probability And Blackjack

The above formula applies to many games of chance. For example, what is the probability that a card drawn at random from deck of playing cards will be an ace? Since the deck has four aces, there are four favorable outcomes; since the deck has 52 cards, there are 52 possible outcomes. The probability is therefore 4/52 = 1/13. What about the probability that the card will be a club? Since there are 13 clubs, the probability is 13/52 = 1/4.

To make this a little more concrete, let's play a little Blackjack. For those who have never played, the object of the game is to get a group of cards (during a single hand of play) whose values come close to or equal 21. If you go over then you automatically lose. However, if you have a better hand (closer or equal to 21) than that of the dealer, you win! Fortunately, you get to do some betting. It's always better when you have some skin in the game. Don't play yet! Read the next paragraph first.

Now, I know that in Vegas, card dealers are working from multiple decks of cards. But as you play a few hands of Blackjack in this interactive simulation, pretend like there is just one 52 card deck that you're dealing with. As you decide whether to *Hit* or *Stand*, think about the information that's available, in numeric terms, and how this leads you to a decision in the game. Are your chances of "busting" on the next card 2/48? How about 46/48? A number of researchers in the field of decision making believe that what you're doing in this game isn't all that different from what you do each day--as you make decisions about what car to buy or whether you should approach your boss about a raise or how you discipline your child. It's all about prediction. What will happen if I do this or that? The big difference between playing cards and making real-world decisions is that the variables involved in making a real-world decision are far more numerous and difficult to define.

Probability As Function Of An Observer's Knowledge

Seems pretty straightforward, right? When we're dealing with theoretical probability, as would be the case with rolling die or pulling cards, it's pretty easy to assign probabilities. Maybe not. Let's re-visit the two categories of probability--theoretical and real-world. As it turns out, the differences between theoretical and real-world probability are not clear cut. How about an example. Let's say you have a bag with 20 cherries, 14 sweet and 6 sour. If you pick a cherry at random, what is the probability that it will be sweet? There are 20 possible cherries that could be picked, so the number of possible outcomes is 20. Of these 20 possible outcomes, 14 are favorable (sweet), so the probability that the cherry will be sweet is 14/20 =7/10. In this example, we're assuming perfect, theoretical probability--all possible outcomes have an equal chance of being selected. In other words, we're assuming that the probability of picking any one of the cherries is the same as the probability of picking any other.

There are potential, real-world complications to this example. What if sweet cherries are smaller than the sour ones or ripe cherries are softer than non-ripe ones. The fact that "touching" the cherries provides additional information about the probability space, changes the probabilities of one or another event occurring. Perhaps the owner of a cheri orchard would be aware of such complications and propose that different probabilities be assigned to the events. This leads us to an important conclusion about probability. As Persi Diaconis (http://en.wikipedia.org/wiki/Persi_Diaconis) has suggested, probability isn't a fact about the world...it's a fact about an observer's knowledge of the world.

What is The Addition ("Or") Rule?

Before we look at the math, let's talk about outcomes that are **mutually exclusive**. Sometimes mutually exclusive events are referred to as *disjoint*. Two events are mutually exclusive if it is *not possible for both of them to occur*. For example, if a die is rolled, the event "getting a 1" and the event "getting a 2" are mutually exclusive since it is not possible for the die to be both a one and a two on the same roll. The occurrence of one event "excludes" the possibility of the other event.

What would be an example of events that are **NOT mutually exclusive**? Consider the two events (1) "It will rain tomorrow in Houston" and (2) "It will rain tomorrow in Galveston (a city near Houston). These events are not mutually exclusive because both of these events can (and often do) occur together.

Probability of Mutually Exclusive Events A or B During a Single Experiment

If Events A and B are mutually exclusive, the probability that either Event A or Event B occurs during a single experiment is:

P(A or B) = P(A) + P(B)

In this discussion we are focusing on a single event. When we say "A or B occurs" we include two possibilities:

1. A occurs and B does not occur
2. B occurs and A does not occur

Now for some examples. *If you flip a coin one time, what is the probability that you will get a head or a tail on that flip?* Letting Event A be a head on the first flip and Event B be a head on the first flip, then P(A) = .5 and P(B) =

.5. Therefore,

P(A or B) = .5 + .5 = 1.0

What we have basically said with this equation is that we're 100% sure that in one toss of the coin, it will land with one side or the other showing. A pretty safe bet, although I remember an episode of *The Twilight Zone* (an old TV show) in which a man flips a coin and it lands on it's side. This leads to all kinds of improbable events in his life.

What about if we roll a six-sided die? *What is the probability of rolling a one or a six on a single role?* Letting Event A be a one and Event B be a six, then P(A) = 1/6 and P(B) = 1/6. Therefore,

P(A or B) = 1/6 + 1/6 = 2/6 which can be reduced to 1/3

Probability of Non-Exclusive Events A or B During a Single Experiment

What if events are not mutually exclusive? Simple. We just do what we did before, only now we build in a subtraction component.

If the events A and B are **not mutually exclusive**, then

p(A or B) = p(A) + p(B) - p(A and B)

The logic behind this formula is that when p(A) and p(B) are added, the occasions on which A and B both occur are counted twice. To adjust for this, p(A and B) is subtracted. *What is the probability that a card selected from a deck will be either an ace or a spade?* The relevant probabilities are:

p(ace) = 4/52
p(spade) = 13/52

The only way in which an ace and a spade can both be drawn is to draw the ace of spades. There is only one ace of spades, so:

p(ace and spade) = 1/52

The probability of an ace or a spade can be computed as:

p(ace) + p(spade)- p(ace and spade) = **4/52 + 13/52 - 1/52 = 16/52 = 4/13**

The Khan Academy (http:/www.khanacademy.org) has a great video that covers this topic. Check it out.

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How are multiple events different from single events?

Independent Events

When we look at the probability of multiple events occurring, we have to begin with a discussion of independent and nonindependent events. Intuitively, two Events A and B are **independent** if the *occurrence of one has no effect on the probability of the occurrence of a subsequent event*. For example, if you throw two dice, the probability that the second one comes up 1 is independent of whether the first role came up 1. Formally, this can be stated in terms of conditional probabilities:

$P(A|B) = P(A)$ and $P(B|A) = P(B)$

The above is read as: *the probability of Event A given that Event B has occurred equals the probability of Event A.* The second part of the probability statement is read as: *the probability of Event B, given that event A has occurred equals the probability of Event B.* In other words, B has no effect on A, and A has no effect on B.

Dependent Events

Conversely, *when one event's occurrence affects the probability of a subsequent event,* we would say that a **dependent** relationship exists between the two events. A nice example of non-independent events is an experiment in which one card is drawn from a deck and not replaced before a second card is drawn. In this case, the probability space shrinks from 52 to 51 cards, thus affecting the probability of the second draw. Here it is stated in conditional probability terms:

$P(B|A) \neq P(B)$

The above is read as: *the probability of Event B, given the previous occurrence of Event A, does not equal the probability of Event B, by itself.*

Deciding whether two events are independent or not can be a very difficult matter and has great implications for our data collection, findings, analysis and ultimately, our decisions. Check out the different examples and explanations of

What is The Multiplication ("And") Rule?

Independent Events in a Series

When two events are *independent*, the probability of both occurring is the *product of the probabilities* of the individual events. More formally, if events A and B are independent, then the probability of both A and B occurring is:

P(A and B) = P(A) x P(B)

where P(A and B) is the probability of events A and B both occurring, P(A) is the probability of event A occurring, and P(B) is the probability of event B occurring. Of course, we can extend this formula to accommodate sequences of more than two desirable events.

If you flip a coin twice, what is the probability that it will come up heads both times? Event A is that the coin comes up heads on the first flip and Event B is that the coin comes up heads on the second flip. Since both P(A) and P(B) equal 1/2, the probability that both events occur is:

1/2 x 1/2 = 1/4 or .5 x .5 = .25

Lets take another example. *If you flip a coin and roll a six-sided die, what is the probability that the coin comes up heads and the die comes up 1?* Since the two events are independent, the probability is simply the probability of a head (which is 1/2) times the probability of the die coming up 1 (which is 1/6). Therefore, the probability of both events occurring is:

1/2 x 1/6 = 1/12

Let's shift gears One final example: You draw a card from a deck of cards, put it back, and then draw another card. *What is the probability that the first card is a heart and the second card is black?* Since there are 52 cards in a deck, and 13 of them are hearts, the probability that the first card is a heart is 13/52 = 1/4. Since there are 26 black cards in the deck, the probability that the second card is black is 26/52 = 1/2. The probability of both events occurring is therefore:

1/4 x 1/2 = 1/8 or **.25 x .5 = .125**

Another way to say this is that 12.5% of the time we run this experiment, we will get the desired events in the series.

Dependent Events in a Series

We are dealing with **non-independent events** when the *occurrence of one event affects the probability of the occurrence of another event*. Non-independent events are also referred to as *conditional*. The probability for these types of events occurring sequentially in a series is expressed as:

$$P(A, B) = P(A) \times P(B|A)$$

The above is read as: *the probability that events A and B will occur is equal to Event A times Event B, given that event A has occurred.* In other words, the occurrence of A has an effect on the probability of B.

For three events, the equation becomes:

$$P(A, B, C) = P(A) \times P(B|A) \times P(C|A, B)$$

Let's go through a quick example. I want to go back to my graduate school days for this one. Suppose that we have *10 guinea pigs crammed into a cage*. Yes. As a graduate student, I did things to guinea pigs that I'm not proud of. Stop judging me!!

Where was I? Oh, yes. Three of the guinea pigs are black, three are white, and four are spotted.

What is the probability of randomly pulling one white and THEN one spotted guinea pig on two consecutive pulls IF we don't put the first one back. Well, the probability of selecting the white one is 3/10 or about 33%. That's the first pull. Now, we said that we're not going to put that cute little guy back. This means that our *sample space* (all possible outcomes) has shrunk by one guinea pig. This always results in a change (lowering) of our denominator. With these conditions in place, the probability of pulling the spotted pig on the second pull is 4/9 or about 44%. Thus, the probability of selecting a white followed by a spotted guinea pig on consecutive draws is:

$$P(W, Sp) = P(W) \times P(Sp|W) = .33 \times .44 = .145$$

If you're having trouble with this, don't just depend on the numbers. Go ahead and DRAW THE PROBLEM. Make some pictures.

Or better, yet--say "Hello" to Statsie--our interactive, class guinea pig and prognosticator of all things gerbil. Because I felt so guilty about my treatment of guinea pigs in college, I adopted Statsie from a wayward school for mathematically challenged rodents. Go ahead and feed Statsie or make her run on the wheel.

Back to work! How about a card example. Suppose we've drawn a 6 from a deck of 52 playing cards and have not replaced it. *What is the probability of our drawing another 6?* The probability of drawing the first 6 was 4/52, or 1/13, or .077, but the probability of drawing a second 6 depends on the first draw. After one 6 has been taken from the deck and not replaced, there are now only three 6s left in a 51 card deck, for a probability of 3/51, or .059. The probability of drawing a pair of 6s in two draws is therefore .077 X .059 = .0045.

If events were not mutually exclusive, we would have to adjust our calculations a little. We won't go into that. Watch the following for more explanation of how to calculate probabilities for dependent events in a series.

Why would we place conditions on probability?

Placing Conditions Improves Our Predictions

Conditional probability analysis *is the process of assessing probabilities of dependent, favorable outcomes by adding information to what we already know.* In other words, it's a process of changing and hopefully, improving the accuracy of our predictions. That's the key--we're adding information as we go through a series of experiments with dependent variables--which of course should alter our proability estimates in a fluid kind of way.

If you think about it, it doesn't sound all that strange--that if we add correct information to what we already know, our understanding of events will improve.

Contingency Tables and Independent/Non-Independent Variables

There are all sorts of scenarios that hinge on conditional probabilities. The *Let's Make a Deal* scenario is merely one application. Just remember that in any conditional probability scenario, information is being added to what is already known, and this new information can change probability estimates and reflexively decisions, judgments and understandings.

A **contingency table** of probabilities provides a different way of assessing conditional probabilities. Moreover, this type of table can help us determine whether certain variables are *independent of* or *dependent upon* one another. The table displays probabilities in relation to two different variables. Later on in this book, we will look at contingency tables again, but in another form. In any event, probabilities are often embedded within tables like this. So, it's important to have at least a little experience interpreting them.

Here is a table depicting some probabilities form the Arts and Sciences College (A&S) at a well-known university. Notice first that in this table we have two nominal-level variables (*Gender* and *Major*). This type of table always comprises categories. Secondly, the numbers in the table represent probabilities and NOT actual frequency counts

(i.e., the number of males and females in
the Psychology and Other Majors). Third,

notice that the totals (along the outside of the table) provide a summation of columns and rows. Finally, the total probability for both variables is 100% or 1.0. The totals in this kind of table always add up to 100% or 1.0. Knowing that the lower, right corner will always add up to 100% allows us to fill in information that might be missing.

	GENDER		
MAJOR	**Male**	**Female**	**Total**
Psychology	.01	.06	.07
Other	.46	.47	.93
Total	.47	.53	1.00

So, what do these data tell us? Among other things, they tell us that the 53% of the students in the A&S are female. Another way to say this is that the probability of randomly selecting a female student in this college is .53 or about 1/2. We also know that only 7% of A&S students are Psychology Majors. So, the probability of randomly selecting an A&S student from another major is .93.

Let's go deeper. *What is the probability of being female AND a psychology student?* The answer is .06 or 6%. Another way we could state our conclusion is that 6 out of every 100 A&S students is female and a Psychology Major. This type of question is easy to answer. When you are asked a question with "and" in it, you know that you will simply be looking at where two columns intersect. In this case *Female* and *Psychology* intersect at .06. Remember...AND = INTERSECTION.

But what if we ask a slightly different question. *GIVEN that a person is majoring in Psychology, what is the probability that this person is female?* The answer to this one is a little more difficult. Let's see. We're saying that we have some information (Psychology Major) and we want to know about some specifics (Gender) within the major. In other words, we are adding information to what is already known. It's the "given" that tips us off. When we see the word *given*, we know that our probability space has shrunk. We are being asked to constrain our sample space to just Psychology Majors. And given just this group of students, what is the probability of selecting a female at random? Here is the math:

P(Female | Psychology Major) = .06/.07 = .86 or 86%

The above is read as: *the probability of being female given that one is majoring in Psychology.*

Interesting. The answer to the first question was .06, but the answer to the second question was .86. That's quite an increase. By constraining our sample space to only Psychology Majors, we've really improved our chances of randomly selecting a female. Because these two numbers are different, we would conclude that *choice of major and gender are dependent on one another or in other words, somehow related*. If Gender and Major were independent of one another, then our responses to the *AND* question and the *GIVEN* question would have been about the same. This example should give you an added appreciation of what conditional probability can add in terms of increasing the accuracy of predictions.

Let's Make a Deal!

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1 [but the door is not opened], and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? Play a few rounds of *Let's Make a Deal* and see if you can figure it out, before moving on.

Don't worry if you've not figured it out. When this scenario was first proposed in a magazine in 1975, and then again in 1990, it baffled thousands of people--including scientists, mathimaticians, Nobel prize winners and even humble psychologists.

So, how many cars did you win? Believe it or not, you will win most often if you switch doors after the host opens one. In fact, your odds of winning are 1/3 if you don't switch doors and 2/3 if you do switch. Most people think that switching (after the host opens the door with the goat behind it) gives them a 50/50 chance of winning, since two doors remain unopened. But such thinking does not take into account a simple fact of conditional probability--after the host opens the door with one of the goats behind it, we have more information than we started with.

Don't worry about whether you understand the numbers behind the *Let's Make a Deal* scenario. The thing that you want to keep in mind is that conditional probability is all about how adding information to what we know can (not always) change probabilities and even improve our decision making.

Extra Credit Application (2 points)

Go out and read up on the "Monty Hall" probability dilemma. You can do a Google (http://www.google.com) or YouTube (http://www.youtube.com) search (for "Monty Hall probability explained" or "Let's Make a Deal probability explained") to find a lot of good explanations of this seemingly counterintuitive experiment. Read until you understand it. Then, answer the two most important questions that a contestant is faced with after he/she selects an initial door and Monty opens another door revealing nothing of value: *1) Should the contestant switch doors? 2) Why?* Your responses must be in your own words. Don't just copy the thoughts of someone else.

On a sheet of NOTEBOOK PAPER, write up your answers to these questions and attach a PRINTOUT of the website that you used. Staple and submit both.

What is a binomial distribution?

This section is a little redundant but I wasn't sure how else to fit it in. When you flip a coin, there are two possible outcomes: heads and tails. A coin flip constitutes a **binomial variable** or event. Each outcome has a fixed probability, the same from trial to trial. In the case of coins, heads and tails each have the same probability of 1/2. But a binomial variable isn't just restricted to a 50/50 probability. More generally, binomial variables comprise probability distributions for which *there are just two possible outcomes with fixed probability summing to one*. Could be 30/70, 40/60, or 20/80. Just so long as the two possible outcomes sum to one. These distributions are called binomial distributions.

Outcome	First Flip	Second Flip
1	Heads	Heads
2	Heads	Tails
3	Tails	Heads
4	Tails	Tails

How about a simple example. The four possible outcomes that could occur if you flipped a coin twice are listed in this table. Note that the four outcomes are equally likely: each has probability 1/4. To see this, note that the tosses of the coin are independent (neither affects the other). Hence, the probability of a head on Flip 1 and a head on Flip 2 is the product of P(H) and P(H), which is 1/2 x 1/2 = 1/4. The same calculation applies to the probability of a head on Flip one and a tail on Flip 2. Each is 1/2 x 1/2 = 1/4.

The four possible outcomes can be classified in terms of the number of heads that come up. The number could be two (Outcome 1), one (Outcomes 2 and 3) or 0 (Outcome 4). The

Number of Heads	Probability
0	1/4

probabilities of these possibilities are shown in this table. Since two of the outcomes represent the case in which just one head appears in the two tosses, the probability of this event is equal to 1/4 + 1/4 = 1/2. This table summarizes the situation.

1	1/2
2	1/4

How Does subjective probability lead To Errors in thinking and decision making?

There are times when our intuitive understanding of probability is incorrect. Overreliance on subjective probabilities can be detrimental. You may know someone who, in spite of good empirical evidence indicating safety, refuses to where her seatbelt. You may also know someone who enter lotteries all of the time, not understanding that the chances of winning are astronomically small. In this section, we're going to talk about some special cases of subjective probability.

Probability=Certainty Fallacy

A lot of people get it in their heads that if someone predicts there is a *better than 50% chance* of something occurring, and that something does not occur, then the prediction was wrong. But here is the mistake in this type of thinking: When we state probabilities, we are talking about the number of times that something will occur *over the long run*. So, concluding that our hypothesis was wrong, based on one experiment, shows a misunderstanding of probability.

Now, if we studied something over the long run, and many trials revealed that there was less than a 50% chance of the event occurring, then we would conclude that the original prediction may not be the best prediction. In other words, when we're talking about probability, it's not a single event that makes or breaks a prediction.

Let's contextualize this. Suppose you wish to know what the weather will be like next Saturday because you are planning a picnic. You turn on your radio, and the weather person says, “There is a 10% chance of rain.” You decide to have the picnic outdoors and, lo and behold, it rains. You are furious with the weather person. But was he or she wrong? No, they did not say it would not rain, only that rain was unlikely. The weather person would have been flatly wrong only if he/she said that the probability is 0 and it subsequently rained. However, if you kept track of the weather predictions over a long periods of time and found that it rained on 50% of the days that the weather person said the probability was 0.10, you could say his or her probability assessments are wrong.

Gambler's Fallacy

Suppose someone offered to bet you that he could flip a coin ten times and it would come up heads every time. He would even let you pick the coin so you could be sure it was a fair coin. What kind of odds would you give him to make the bet? 100-to-1? 1,000-to-1?

Now what if he flipped the coin nine times and it came up heads every time? For the tenth flip, what would your thinking be? Would you think that after so many flips coming up the same, he is due for a different result? Would you give him the same 1,000-to-1 against heads? If you were to do so, it would be a big mistake, and this mistake is the cornerstone of the gambler's fallacy.

The **gambler's fallacy** is the *mistaken idea that previous trials affect the odds of later trials*. This applies almost exclusively to *independent events* based on *theoretical probability*. You can read about those in another section of this book. Now, in the above example, the fact that heads showed up nine times in a row has absolutely no bearing on whether the coin will come up heads the tenth time. Once the nine have already happened, the odds of the tenth flip coming up heads are still 50-50--just like the first flip. The coin has no memory and you might as well be flipping it the first time, or be flipping a different coin.

We see the Gambler's Fallacy applied in real life all the time. Examples include:

- Choosing lottery numbers based on past weeks' results ("the number 14 hasn't come up in two months!")
- "My neighbour's 15-year-old kid had a hole-in-one last week! He better enjoy it, since it's a once in a lifetime thing and he's just used up his!"
- Setting up your fantasy football team ("Jay Feely lead all kickers in scoring last year. No kicker has lead in back-to-back years since '79/'80. So, there's no chance Feely could do it again.")
- Continuing to pull the lever on a slot machine time after time ("Hey...since it hasn't paid off in a while, I'm due!")

Let's talk about this last one and introduce a little behavioral psychology. While most theorists explain the gambler's fallacy as strictly a fudging of true probability in one's head, it's more than that. Think about someone playing a slot machine. The reward for such an event happens on average after a certain number of pulls. Because this "on average" payoff governs the game, each successive pull of the lever induces a greater and greater feeling that reward is even more imminent than it was after the last pull. In fact, the odds have not changed--only your gut feeling and the cognitive, subjective butchering of true probability that occurs when placed in a situation like this.

Availability Heuristic

The **availability heuristic** is a type of thinking in which *people estimate the probability of an event based on how easily an example can be brought to mind*. Images are easier to bring to mind when they are personal, repeated, emotionally powerful and recent.

Availability ...

Essentially the availability heuristic operates on the notion that "if you can think of it, it must be important and likely to happen." Media coverage can help fuel a person's example bias with widespread and extensive coverage of unusual events, such as homicide or airline accidents, and less coverage of more routine, less sensational events, such as common diseases or car accidents. For example, when asked to rate the probability of a variety of causes of death, people tend to rate more "newsworthy" events as more likely because they can more readily recall an example from memory. For example, in the USA, people rate the chance of death by homicide higher than the chance of death by stomach cancer, even though death by stomach cancer is five times higher than death by homicide. Moreover, unusual and vivid events like homicides, shark attacks, or lightning are more often reported in mass media than common and unsensational causes of death like common diseases.

Politicians often use the availability heuristic to further their own agendas. A senator or congress person can get people to vote for his/her policies by discussing or showing pictures of powerful emotional events that suggest a strong need for such policies. Often times, such tactics are used to invoke irrational fears among voters. Think about how much more positively voters will view the re-construction of an old bridge, if images, sounds and stories of horrific bridge collapses are repeatedly presented, right before a crucial vote.

Self Test

- [Self-test for chapter (self_test_probability.pdf)]

- Answers to self-test (answers_probability.pdf)

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Some content adapted from other's work. See home page for specifics.

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