Comparing Two Group Averages

The Two-Sample t-Test

Overview

- Variations of t-tests
- Directional vs. non-directional tests
- Signal vs. noise
- Sampling distribution of mean differences
- Independent groups t-test
- Effect size tests
- Dependent groups t-test

Variations of t tests

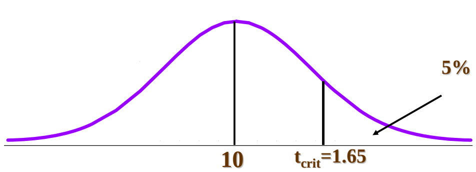
- One-sample t-test: Used when comparing sample to population mean and population mean is known.
- Independent: When population mean is unknown, comparisons may be made between treatment and control groups.
- Dependent: When population mean is unknown, comparisons may be made between paired groups.

Directional vs. Non-Directional Tests

An ethical consideration?

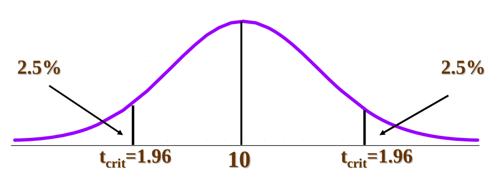
One-Tailed Test

- Directional: When predicting which sample mean will be higher/lower.
- Rejection region located in only one tail
- LESS stringent critical t.



Two-Tailed Test

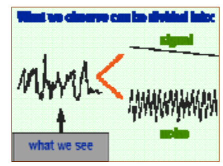
- Non-directional: Does not predict which group mean will be higher/lower
- Rejection region distributed between two tails
- MORE stringent critical t

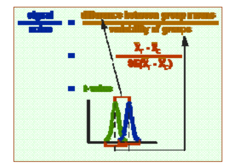


T-Tests, Signals, Noises & Things That Go Bump In Your Data...

Metaphor: signal and noise

- Signal: Difference between means
- Noise: Variability within groups

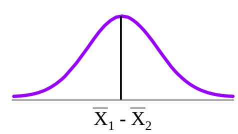




Independent Groups t-Test

Sampling distribution of mean differences

- Mean differences plotted on x axis
- Mean is 0 if two group means equal one another
- Larger sample sizes lead to more normal distribution
- Larger sample sizes reduce standard error of mean differences…



This comparison…	is equivalent to …
$\mu_1 \neq \mu_2$	$\mu_1 - \mu_2 \neq 0$
$\mu_1 > \mu_2$	$\mu_1 - \mu_2 > 0$
$\mu_1 < \mu_2$	$\mu_1 - \mu_2 < 0$

Estimated Standard Error Of The Mean Difference

$$s_{\bar{X}_1-\bar{X}_2} = \sqrt{\left[\frac{(N_1-1)s_1^2+(N_2-1)s_2^2}{N_1+N_2-2}\right]\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}$$

Say it! 

- Turn to your neighbor and express these equations in real words and not this cryptic statistical symbolic nonsense!

$$s = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N-1}}$$

$$s_{\bar{X}_1-\bar{X}_2} = \sqrt{\left[\frac{(N_1-1)s_1^2+(N_2-1)s_2^2}{N_1+N_2-2}\right]\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}$$

Practice

- For each of the following, compute the estimated standard error of the difference between means--using both the raw-score and the defining formulas. Assume that the samples are independent.

  a. $N_1 = 12$, $N_2 = 12$, $s_1 = 3.6$, $s_2 = 4.3$
  b. $N_1 = 22$, $\sum X_1 = 112.2$, $\sum X_1^2 = 643.5$; $N_2 = 22$, $\sum X_2 = 138.6$, $\sum X_2^2 = 1{,}010.68$

$$s_{\bar{X}_1-\bar{X}_2} = \sqrt{\left[\frac{(N_1-1)s_1^2+(N_2-1)s_2^2}{N_1+N_2-2}\right]\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}$$

How to compute t

$$t = \frac{\text{Difference between sample means}}{\text{Standard error of the difference}}$$

$$t_{\bar{X}_1-\bar{X}_2} = \frac{\bar{X}_1-\bar{X}_2}{s_{\bar{X}_1-\bar{X}_2}}$$

Expanded Formula

$$t_{\bar{X}_1-\bar{X}_2} = \frac{\bar{X}_1-\bar{X}_2}{\sqrt{\left[\frac{(N_1-1)s_1^2+(N_2-1)s_2^2}{N_1+N_2-2}\right]\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}}$$

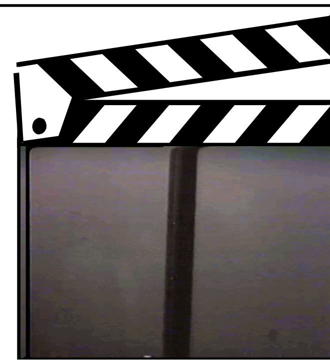
•Remember that your degrees of freedom is based on $N_1 + N_2 - 2$.

Independent t-Test Assumptions

- Sample data are normally distributed
- Variances of two groups are about equal or homogeneous
- Scores in the two groups are independent (i.e., don't depend on one another)

Same Basic Steps

1. Set up Null and alternative hypotheses:
   H_0: $\mu_1 = \mu_2$
   H_1: $\mu_1 \neq \mu_2$
2. Set alpha and critical t
3. Collect data
4. Crunch the numbers (ensuring that assumptions are met)
5. Draw conclusions via comparison of computed t with critical t. Decide whether to accept or reject. Restate your conclusions in everyday language. Throw in a probability statement.



Improving the Welfare System

LOOK CLOSER!

- Determine for yourself what advice should be given to congress. Are *Options* graduates making significantly more money than grads from the traditional program? Data represent average income for 12 hours of work.

Options	Traditional
N = 25	*N* = 25
Mean = 81.7	Mean = 71.4
s = 8.3	*s* = 10.1

$$t_{\bar{X}_1-\bar{X}_2} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left[\frac{(N_1-1)s_1^2+(N_2-1)s_2^2}{N_1+N_2-2}\right]\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}}$$

Effect Size

A test of practical significance

What the t-statistic means

- Statistical significance is not practical significance
  - On the next slide, pay close attention to three entries: sample size, mean difference and probability level (p).
  - What do you notice?

What is the effect of time on memory recall?

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Paired T for 1hour - 24hour

                    N      Mean     StDev   SE Mean
1hour             100     12.75      3.69      1.31
24hour            100     11.43      3.52      1.25
Difference        100      1.32     2.066
0.730

95% CI for mean difference: (3.897, 5.353)
T-Test of mean difference = 0 (vs not

T-Value = 2.66
P-Value = 0.01
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Is the mean difference big enough?

Cohen's *d*

- Statistical differences are not always practically meaningful
- Large vs. small n problem
- Cohen's *d* supplements hypothesis tests with analysis of practical significance
  - $(M_1 - M_2)/S$
    - If S for two groups approx equal, S derives from pooled standard deviation: $(S_1+S_2)/2$

LOOK CLOSER!

- Use Cohen's *d* formula to determine whether the earnings difference between *OPTIONS* and *TRADITIONAL* grads is practically meaningful:
  - Options: M=81.7, s=8.3
  - Traditional: M=71.4, s=10.1

Cohen's *d* Significance Criteria

- Small effect = .2
- Medium effect = .5
- Large effect = .8

Cohen's *d*:$(M_1 - M_2)/S$

LOOK CLOSER!

- Now what do you think? Forget all of the numbers that we just calculated. How could you intuitively figure out whether the difference between the groups has practical value?

More Practice With Independent t

Practice

- In a military training program, the complex reaction-time ability of pilots and navigators is compared. Determine whether there is a significant difference statistically between the groups in the number of errors made (failure to respond) in 100 stimulus presentations.

Pilots	Navigators
N = 15	*N* = 20
Mean = 23.5	Mean = 41.3
s = 10.5	*s* = 12.7

$$t_{\bar{X}_1-\bar{X}_2} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left[\frac{(N_1-1)s_1^2 + (N_2-1)s_2^2}{N_1+N_2-2}\right]\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}}$$

Practice

- A study is done to see whether children are more likely to have psychic powers than young adults. Thirteen randomly selected 8-year-old children each make one run through a standard PSI deck trying to guess the identity of each target card by "reading" the mind of the tester. There are four different targets, and each child receives a score indicating proportion of "hits." The same procedure is used on 15 randomly selected college students. Compare the groups.

Children	Young Adults
N = 13	N = 15
mean = .28	mean = .24
s = .067	s = .073

Practice

- In an attempted replication of the Rosenthal effect (experimenter bias can influence the outcome of an experiment), two groups of 15 randomly selected students each are given some rats to train. One group is told that the rats are unintelligent, whereas the other group is told that the rats are intelligent. Errors are recorded during training, and the data for each group are shown here.

Group "Stupid"	Group "Intelligent"
$\sum X$ = 379.5	$\sum X$ = 252
$\sum X^2$ = 9,853.5	$\sum X^2$ = 4,450.2
N = 15	*N* = 15

a. Before you analyze the data, determine whether there is a rationale for using the one-tailed test of significance. If so, what is it?

b. Compare the groups using the appropriate statistical test.

OMITTED SLIDES

Probability Level: 3 Ways to Say It

1. **"How likely is it that our sample means would differ by as much as X if the difference in population means really is 0?"**
2. **"How likely is it that our sample means would be this different if the two samples came from the same population?"**
3. **"How likely is it that our sample means would be this different due to chance alone?"**

Reflection Moment

APA STYLE

ACCEPT or REJECT? WHY?

$t(15) = 1.2$, $p < .1$

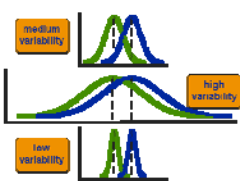
$t(60) = 2.6$, $p > .001$

$t(45) = 3.8$, $p < .05$

$t(80) = -1.7$, $p > .01$

What the t-test does

- **We look at the differences between two means relative to the spread or variability of group scores.**



Reflection Moment

- **Here are some hypotheses about online communication. What might the signals be in these hypotheses? How about noise? In other words, what problems might you encounter in the process of operationalization?**
  1. **Some people experience their message as a piece of themselves.**
  2. **Even though we may not be fully aware of it, we always develop a mental image of the other person in a text relationship.**
  3. **Humor, and especially sarcasm, is difficult to express in text relationships.**
  4. **Text relationships can be used to desensitize social anxieties and build social skills.**

Draw it!

- **Draw the two sample curves next to one another, with accurate overlap and variability.**
  a. $Mean_1 = 12$, $mean_2 = 16$, $s_1 = 3.6$, $s_2 = 4.3$
  b. $N_1 = 22$, $\sum X_1 = 112.2$, $\sum X_1^2 = 643.5$; $N_2 = 22$, $\sum X_2 = 138.6$, $\sum X_2^2 = 1{,}010.68$